Ecole d'été
Mécanique Théorique
Quiberon

septembre 2015

Cours 5

Effets non locaux

$$D_{ij} = D^e_{ij} + D^p_{ij}$$

élasticité $\hat{\sigma}_{ij} = C_{ijkl}(D_{kl} - D^p_{kl})$

Fonction de charge $f(\boldsymbol{\sigma}, R) \leq 0$

Loi d'écoulement $D^p_{ij} = \lambda \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$

Loi d'écrouissage $\dot{p} = \lambda \dfrac{\partial f}{\partial R}$

$$\lambda \geq 0 \ , \ f \leq 0 \text{ and } \lambda \dot{f} = 0$$

Un exemple simple

Cadre Local

$$\dot{\sigma} = \mathbf{E} : (\dot{\varepsilon} - \dot{\varepsilon}^{\mathrm{p}}).$$

$$f[\sigma, R(p)] = 0$$

$$\dot{\varepsilon}^{\mathrm{p}} = \lambda \frac{\partial F}{\partial \sigma}$$

$$\dot{p} = \lambda \frac{\partial F}{\partial R}$$

$$\lambda \geqslant 0, \lambda \dot{f} = 0, \quad f \leqslant 0$$

Cadre non local

$$\dot{\sigma} = \mathbf{E} : (\dot{\varepsilon} - \dot{\varepsilon}^{\mathrm{p}}).$$

$$f = f(\boldsymbol{\sigma}, \boldsymbol{p}, \boldsymbol{\Delta p}).$$

$$\dot{\varepsilon}^{\mathrm{p}} = \lambda \frac{\partial F}{\partial \sigma}$$

$$\dot{p} = \lambda \frac{\partial F}{\partial R}$$

$$\lambda \geqslant 0, \lambda \dot{f} = 0, \quad f \leqslant 0$$

$$\dot{p} = \sqrt{2\dot{\varepsilon}^{\mathrm{p}} : \dot{\varepsilon}^{\mathrm{p}}}.$$

Evolution du multiplicateur plastique

$$\lambda = \frac{\frac{\partial f}{\partial \sigma} : \mathbf{E} : \dot{\varepsilon}}{h + \frac{\partial f}{\partial \sigma} : \mathbf{E} : \frac{\partial F}{\partial \sigma}},$$

$$h = -\frac{\partial f}{\partial R}\frac{\partial R}{\partial p}\frac{\partial F}{\partial R}$$

$$\frac{\partial f}{\partial \sigma} : \mathbf{E} : \dot{\varepsilon} - H\lambda + \bar{\omega}(\text{grad}\lambda).\left(\text{grad}\frac{\partial F}{\partial R}\right) + \omega\Delta\lambda = 0$$

$$H = h + \frac{\partial f}{\partial \sigma} : \mathbf{E} : \frac{\partial F}{\partial \sigma} + \frac{\partial f}{\partial R}\frac{\partial R}{\partial(\Delta p)}\Delta\left(\frac{\partial F}{\partial R}\right)$$

$$\bar{\omega} = 2\frac{\partial f}{\partial R}\frac{\partial R}{\partial(\Delta p)}$$

$$\omega = \frac{\partial f}{\partial R}\frac{\partial R}{\partial(\Delta p)}\frac{\partial F}{\partial R}$$

$$\omega = \alpha l^2, \quad \bar{\omega} = \beta l,$$

Extension grandes transformations

$$\hat{\sigma} = \dot{\sigma} - \mathbf{\Omega}.\sigma + \sigma.\mathbf{\Omega} = \mathbf{E}:(\mathbf{D} - \mathbf{D}^{\mathrm{p}})$$

Incompressibilité

$$\hat{\sigma} = \mathbf{E}:(\mathbf{D} - \mathbf{D}^{\mathrm{p}}) + \hat{g}\mathbf{1}$$

$$\frac{\partial f}{\partial \sigma} : \mathbf{E} : \mathbf{D} - H\lambda + \omega\Delta\lambda = 0$$

$$\frac{\partial f}{\partial \sigma} : \mathbf{E} : \mathbf{D} + \mathrm{tr}\left(\frac{\partial f}{\partial \sigma}\right)\hat{g} - H\lambda + \omega\Delta\lambda = 0$$

$$\dot{\mathbf{S}} = \dot{\sigma} - \mathbf{V} \cdot \sigma + \sigma \operatorname{tr}(\mathbf{D})$$

$$\dot{\mathbf{S}} = (\mathbf{E} + \mathbf{A}) : \mathbf{V} - \lambda \mathbf{E} : \frac{\partial F}{\partial \sigma},$$

$$\dot{\mathbf{S}} = (\mathbf{E} + \mathbf{B}) : \mathbf{V} - \lambda \mathbf{E} : \frac{\partial F}{\partial \sigma} + \hat{g}\mathbf{1}$$

$$\mathbf{A}_{ijkl} = -\tfrac{1}{2}\delta_{ik}\sigma_{lj} - \tfrac{1}{2}\delta_{il}\sigma_{kj} - \tfrac{1}{2}\delta_{jk}\sigma_{il} + \tfrac{1}{2}\delta_{jl}\sigma_{ik} + \delta_{kl}\sigma_{ij}$$

$$\mathbf{B}_{ijkl} = -\tfrac{1}{2}\delta_{ik}\sigma_{lj} - \tfrac{1}{2}\delta_{il}\sigma_{kj} - \tfrac{1}{2}\delta_{jk}\sigma_{il} + \tfrac{1}{2}\delta_{jl}\sigma_{ik}.$$

Probleme en vitesse

$$\operatorname{div}\left\{[(\mathbf{E}+\mathbf{A}):\mathbf{V}]^{\mathrm{T}}-\lambda\mathbf{E}:\frac{\partial F}{\partial\sigma}\right\}=\mathbf{0},$$

$$\frac{\partial f}{\partial\sigma}:\mathbf{E}:\mathbf{D}-H\lambda+\omega\Delta\lambda=0,$$

$$\lambda\geqslant 0\,,\lambda\dot{f}=0\,,\quad f\leqslant 0$$

$$\lambda=0\quad\text{or}\quad\frac{\partial\lambda}{\partial\mathbf{n}}=0$$

+ CL en vitesses et efforts

Cas incompressible

$$\mathrm{div}\left\{[(\mathbf{E}+\mathbf{B}):\mathbf{V}]^{\mathrm{T}} - \lambda\mathbf{E}:\frac{\partial F}{\partial \sigma} + \hat{g}\mathbf{1}\right\} = \mathbf{0}$$

$$\frac{\partial f}{\partial \sigma}:\mathbf{E}:\mathbf{D} + \mathrm{tr}\left(\frac{\partial f}{\partial \sigma}\right)\hat{g} - H\lambda + \omega\Delta\lambda = 0,$$

$$\mathrm{tr}(\mathbf{D}) = 0.$$

$$\lambda \geqslant 0\,,\ \lambda\dot{f} = 0\,, \quad f \leqslant 0$$

$$\lambda = 0 \quad \text{or} \quad \frac{\partial \lambda}{\partial \mathbf{n}} = 0$$

+ CL en vitesses et efforts

Bifurcation

Si 2 solutions distinctes

$$I(\mathbf{v}_1, \mathbf{v}_2, \lambda_1, \lambda_2) = \int_\Omega (\mathbf{V}_1^{\mathrm{T}} - \mathbf{V}_2^{\mathrm{T}}) : (\dot{\mathbf{S}}_1 - \dot{\mathbf{S}}_2)\, \mathrm{d}v = 0,$$

$$I(\mathbf{v}_1, \mathbf{v}_2, \lambda_1, \lambda_2) = \int_\Omega \left\{ (\mathbf{V}_1^{\mathrm{T}} - \mathbf{V}_2^{\mathrm{T}}) : \mathbf{E} : (\mathbf{V}_1 - \mathbf{V}_2) - (\lambda_1 - \lambda_2)(\mathbf{V}_1^{\mathrm{T}} - \mathbf{V}_2^{\mathrm{T}}) : \mathbf{E} : \frac{\partial F}{\partial \sigma} \right\} \mathrm{d}v$$

$$+ \int_\Omega (\mathbf{V}_1^{\mathrm{T}} - \mathbf{V}_2^{\mathrm{T}}) : \mathbf{A} : (\mathbf{V}_1 - \mathbf{V}_2)\, \mathrm{d}v = 0,$$

Critère de non-bifurcation

$$\boldsymbol{I}(\mathbf{v}_1, \mathbf{v}_2, \lambda_1, \lambda_2) > 0$$

Solide linéaire de comparaison

$$\hat{\lambda}(\mathbf{w}) = \int_{\Omega_p} G(\mathbf{x}, \mathbf{y}) \frac{\partial f}{\partial \sigma} : \mathbf{E} : \mathbf{D}(\mathbf{w}) \, \mathrm{d}y.$$

$$H(\mathbf{v}) = \int_{\Omega} \mathbf{D}(\mathbf{v}) : \mathbf{E} : \mathbf{D}(\mathbf{v}) \, \mathrm{d}\mathbf{x} - \int_{\Omega_p} \left\{ \int_{\Omega_p} G(\mathbf{x}, \mathbf{y}) \frac{\partial f}{\partial \sigma} : \mathbf{E} : \mathbf{D}(\mathbf{v}(\mathbf{y})) \, \mathrm{d}\mathbf{y} \right\} \left[\mathbf{D}(\mathbf{v}(\mathbf{x})) : \mathbf{E} : \frac{\partial f}{\partial \sigma} \right] \mathrm{d}x$$

$$+ \int_{\Omega} (\mathbf{V}_1^{\mathrm{T}} - \mathbf{V}_2^{\mathrm{T}}) : \mathbf{A} : (\mathbf{V}_1 - \mathbf{V}_2) \, \mathrm{d}v$$

$$I(\mathbf{v}_1, \mathbf{v}_2, \lambda_1, \lambda_2) \quad > \quad H(\mathbf{v})$$

Local vs non local

$$H(\mathbf{v}) = \int_\Omega \left\{ \mathbf{D}(\mathbf{v}) : \mathbf{E} : \mathbf{D}(\mathbf{v}) - (\lambda_1 - \lambda_2)\mathbf{D}(\mathbf{v}) : \mathbf{E} : \frac{\partial F}{\partial \sigma} \right\} \mathrm{d}x + \int_\Omega \mathbf{V}^\mathrm{T} : \mathbf{A} : \mathbf{V}\, \mathrm{d}x > 0.$$

$$\lambda(\mathbf{v}) = \lambda_1 - \lambda_2 = \frac{1}{H} \left\{ \omega \Delta\lambda(\mathbf{v}) + \frac{\partial f}{\partial \sigma} : \mathbf{E} : \mathbf{D}(\mathbf{v}) \right\}.$$

$$H(\mathbf{v}) = \int_\Omega \left\{ \mathbf{D}(\mathbf{v}) : \mathbf{E} : \mathbf{D}(\mathbf{v}) - \frac{1}{H} \left[\mathbf{D}(\mathbf{v}) : \mathbf{E} : \frac{\partial f}{\partial \sigma} \right]^2 \right\} \mathrm{d}x - \int_{\Omega_\mathrm{p}} \frac{1}{H} \left[\mathbf{D}(\mathbf{v}) : \mathbf{E} : \frac{\partial f}{\partial \sigma} \right] \mathrm{d}x$$

$$+ \int_\Omega \mathbf{V}^\mathrm{T} : \mathbf{A} : \mathbf{V}\, \mathrm{d}x - \int_{\Omega_\mathrm{p}} \frac{\omega}{H} \Delta\lambda \left[\mathbf{D}(\mathbf{v}) : \mathbf{E} : \frac{\partial f}{\partial \sigma} \right] \mathrm{d}x.$$

Local vs non local

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{E} : \mathbf{D}(\mathbf{v}) = H\lambda - \omega \Delta \lambda,$$

$$H(\mathbf{v}) = \int_\Omega \left\{ \mathbf{D}(\mathbf{v}) : \mathbf{E} : \mathbf{D}(\mathbf{v}) - \frac{1}{H} \left[\mathbf{D}(\mathbf{v}) : \mathbf{E} : \frac{\partial f}{\partial \boldsymbol{\sigma}} \right]^2 \right\} \mathrm{d}x + \int_\Omega \mathbf{V}^{\mathrm{T}} : \mathbf{A} : \mathbf{V} \,\mathrm{d}x$$
$$+ \int_{\Omega_{\mathrm{p}}} \frac{\omega^2}{H} (\Delta \lambda)^2 \,\mathrm{d}x - \int_{\Omega_{\mathrm{p}}} \omega \lambda \Delta \lambda \,\mathrm{d}x$$

$$\int_{\Omega_{\mathrm{p}}} \omega \lambda \Delta \lambda \,\mathrm{d}x = -\int_{\Omega_{\mathrm{p}}} \omega [\mathrm{grad}\, \lambda]^2 \,\mathrm{d}x + \int_{\partial \Omega_{\mathrm{p}}} \omega \lambda \frac{\partial \lambda}{\partial \mathbf{n}} \,\mathrm{d}s$$

Local vs non local

$$H(\mathbf{v}) = F(\mathbf{v}) + \int_{\Omega_p} \frac{\omega^2}{H}(\Delta\lambda)^2 \, \mathrm{d}x + \int_{\Omega_p} \omega[\operatorname{grad} \lambda]^2 \, \mathrm{d}x,$$

$$F(\mathbf{v}) = \int_{\Omega} \left\{ \mathbf{D}(\mathbf{v}) : \mathbf{E} : \mathbf{D}(\mathbf{v}) - \frac{1}{H}\left[\mathbf{D}(\mathbf{v}) : \mathbf{E} : \frac{\partial f}{\partial \sigma} \right]^2 \right\} \mathrm{d}x + \int_{\Omega} \mathbf{V}^{\mathrm{T}} : \mathbf{A} : \mathbf{V} \, \mathrm{d}x$$

Première bifurcation locale précède toujours celle du milieu non local

Essai de traction/ compression plane

Relations de comportement

$$\mathbf{E}:\frac{\partial f}{\partial \sigma}=\begin{bmatrix} f_{11} & 0 & 0 \\ 0 & f_{22} & 0 \\ 0 & 0 & f_{33} \end{bmatrix} \quad \text{and} \quad \mathbf{E}:\frac{\partial F}{\partial \sigma}=\begin{bmatrix} F_{11} & 0 & 0 \\ 0 & -F_{11} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$\hat{\sigma}_{11} = (E_{1111}-E_{1122})D_{11} - \lambda F_{11} + \hat{g},$$

$$\hat{\sigma}_{22} = (E_{2222}-E_{2211})D_{22} + \lambda F_{11} + \hat{g},$$

$$\hat{\sigma}_{12} = E_{1212}D_{12},$$

$$2\mu^* = E_{1111} - E_{1122} - \frac{F_{11}(f_{11}-f_{22})}{H}$$

$$2\mu = E_{1212},$$

$$\delta = \frac{F_{11}\,\mathrm{tr}\left(\frac{\partial f}{\partial \sigma}\right)}{H}.$$

$$\hat{\sigma}_{11} = 2\mu^* D_{11} + (1-\delta)\hat{g},$$

$$\hat{\sigma}_{22} = 2\mu^* D_{22} + (1+\delta)\hat{g},$$

$$\hat{\sigma}_{12} = 2\mu D_{12},$$

Incompressibilité

$$D_{11} + D_{22} = \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0$$

$$v_1 = \frac{\partial \Psi}{\partial x_2} \text{ and } v_2 = -\frac{\partial \Psi}{\partial x_1}.$$

$$\dot{S}_{11} = \hat{\sigma}_{11} - V_{11}\sigma_{11} = (2\mu - \sigma)\frac{\partial^2\Psi}{\partial x_1\,\partial x_2} - \lambda F_{11} + \hat{g},$$

$$\dot{S}_{22} = \hat{\sigma}_{22} - V_{22}\sigma_{22} = -2\mu\frac{\partial^2\Psi}{\partial x_1\,\partial x_2} - \lambda F_{22} + \hat{g},$$

$$\dot{S}_{12} = \hat{\sigma}_{12} + \Omega_{12}(\sigma_{22} - \sigma_{11}) - V_{12}\sigma_{22} = (\mu - \tfrac{1}{2}\sigma)\frac{\partial^2\Psi}{\partial x_2^2} - (\mu + \tfrac{1}{2}\sigma)\frac{\partial^2\Psi}{\partial x_1^2}$$

$$\dot{S}_{21} = \hat{\sigma}_{21} + \Omega_{12}(\sigma_{22} - \sigma_{11}) - V_{21}\sigma_{11} = (\mu - \tfrac{1}{2}\sigma)\frac{\partial^2\Psi}{\partial x_2^2} - (\mu - \tfrac{1}{2}\sigma)\frac{\partial^2\Psi}{\partial x_1^2}$$

conditions aux limites

$x_1 = \pm a_1$

$$v_1 = \frac{\partial \Psi}{\partial x_2} = 0\,, \quad \dot{S}_{12} = 0\,, \quad \frac{\partial \lambda}{\partial x_1} = 0$$

$x_2 = \pm a_2$

$$\dot{S}_{21} = 0\,, \quad \dot{S}_{22} = 0\,, \quad \frac{\partial \lambda}{\partial x_2} = 0.$$

Solutions

$$\Psi(x_1, x_2) = \cos(\gamma x_1)\psi(x_2)$$

$$\lambda(x_1, x_2) = \sin(\gamma x_1)\varphi(x_2)$$

$$\hat{g}(x_1, x_2) = \sin(\gamma x_1)\rho(x_2).$$

$$\gamma = \gamma_m = \frac{(2m+1)\pi}{2a_1}, \quad m = 1, 2, \ldots$$

Solution

$$\mathbf{A}\cdot\frac{d^3\mathbf{Z}}{\mathrm{d}x_2^3}+\mathbf{B}\cdot\frac{d^2\mathbf{Z}}{\mathrm{d}x_2^2}+\mathbf{C}\cdot\frac{d\mathbf{Z}}{\mathrm{d}x_2}+\mathbf{D}\cdot\mathbf{Z}=0$$

$$\mathbf{Z}(x_2)=\begin{bmatrix}\psi(x_2)\\ \varphi(x_2)\\ \rho(x_2)\end{bmatrix}$$

$$\mathbf{A}=\begin{bmatrix}\mu-\frac{1}{2}\sigma & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}0 & 0 & 0\\ (\mu+\frac{1}{2}\sigma)\gamma_m & 0 & 0\\ 0 & \omega & 0\end{bmatrix}\quad \mathbf{C}=\begin{bmatrix}-(\mu-\frac{1}{2}\sigma)\gamma_m^2 & 0 & 0\\ 0 & F_{11} & 1\\ -(f_{11}-f_{22})\gamma_m & 0 & 0\end{bmatrix},\quad \mathbf{D}=\begin{bmatrix}0 & -F_{11}\gamma_m & \gamma_m\\ -(\mu+\frac{1}{2}\sigma)\gamma_m^3 & 0 & 0\\ 0 & -H-\omega\gamma_m^2 & \mathrm{tr}\left(\frac{\partial f}{\partial\sigma}\right)\end{bmatrix}$$

Solution

$$\mathbf{Z}(x_2) = \exp(r\gamma_m x_2)\mathbf{R}$$

$$\{r^3\gamma_m^3\mathbf{A} + r^2\gamma_m^2\mathbf{B} + r\gamma_m\mathbf{C} + \mathbf{A}\} \cdot \mathbf{R} = \mathbf{0}$$

$$\det\{r^3\gamma_m^3\mathbf{A} + r^2\gamma_m^2\mathbf{B} + r\gamma_m\mathbf{C} + \mathbf{A}\} = 0$$

$$\omega\gamma_m^2(\mu - \tfrac{1}{2}\sigma)r^6 - [(\mu - \tfrac{1}{2}\sigma)(1+\delta)H + \omega\gamma_m^2(3\mu - \tfrac{1}{2}\sigma)]r^4$$
$$+ [(4\mu^* - 2\mu - \sigma\delta)H + \omega\gamma_m^2(3\mu + \tfrac{1}{2}\sigma)]r^2 - (\mu + \tfrac{1}{2}\sigma)[(1-\delta)H + \omega\gamma_m^2] = 0$$

$$(\mu - \tfrac{1}{2}\sigma)(1+\delta)r^4 - (4\mu^* - 2\mu - \sigma\delta)r^2 + (\mu + \tfrac{1}{2}\sigma)(1-\delta) = 0$$

Equation caractéristique

$$\Sigma = \frac{\sigma}{2\mu},$$

$$\varepsilon_m = \frac{\omega\gamma_m^2}{H} = \frac{(2m+1)^2\pi^2\alpha}{H}\left(\frac{l}{a_1}\right)^2$$

$$\varepsilon_m = (\gamma_m^2 a_2^2)\frac{\omega}{Ha_2^2} = (\gamma_m^2 a_2^2)\frac{\alpha l^2}{Ha_2^2}$$

$$\chi = \frac{\alpha l^2}{Ha_2^2}$$

$$(1-\Sigma)r^6 - [(1-\Sigma)(1+\delta)+(3-\Sigma)\varepsilon_m]r^4 + \left[\left(\frac{4\mu^*}{\mu} - 2 - \Sigma\delta\right) + (3+\Sigma)\varepsilon_m\right]r^2 - (1+\Sigma)[(1-\delta)+\varepsilon_m] = 0.$$

$$\mathbf{Z}(x_2) = \sum_{i=1}^{6} \exp(r_i\gamma_m x_2)\mathbf{R}_i$$

Ellipticité

$$\det \mathsf{A}(\mathbf{N}) = 0 \qquad\qquad \mathbf{N} = \gamma \mathbf{n},$$

$$\mathsf{A}(\mathbf{N}) = \begin{bmatrix} \mathbf{N}\cdot(\mathbf{E}+\mathbf{A})\cdot\mathbf{N} & \mathbf{0} \\ \mathbf{0} & \omega(\mathbf{N}\cdot\mathbf{N}) \end{bmatrix}$$

$$\omega(\mathbf{N}\cdot\mathbf{N}) \det [\mathbf{N}\cdot(\mathbf{E}+\mathbf{A})\mathbf{N}] = 0$$

Singularité du tenseur acoustique élastique $\quad |\sigma/2\mu| \geqslant 1$

Localisation

$$\mathbf{v}(\mathbf{x}) = \mathbf{v}_0 \exp(\mathrm{i}\gamma \mathbf{n} \cdot \mathbf{x}), \quad \lambda(\mathbf{x}) = \lambda_0 \exp(\mathrm{i}\gamma \mathbf{n} \cdot \mathbf{x})$$

$$\left[\mathbf{n} \cdot (\mathbf{E} + \mathbf{A}) \cdot \mathbf{n} - \frac{\mathbf{n} \cdot \mathbf{E} : \dfrac{\partial F}{\partial \sigma} \otimes \dfrac{\partial f}{\partial \sigma} : \mathbf{E} \cdot \mathbf{n}}{H + \omega \gamma^2} \right] \cdot \mathbf{v}_0 = \mathbf{0}$$

$$\lambda_0 = \frac{1}{H + \omega \gamma^2} \left(\frac{\partial f}{\partial \sigma} : \mathbf{E} \cdot \mathbf{n} \right)^{\mathrm{T}} \cdot \mathbf{v}_0 .$$

$$\det(\mathbf{n} \cdot \mathbf{H}^*(\gamma) \cdot \mathbf{n}) = 0$$

$$\mathbf{H}^*(\xi) = \mathbf{E} + \mathbf{A} - \frac{\mathbf{E} : \dfrac{\partial F}{\partial \sigma} \otimes \dfrac{\partial f}{\partial \sigma} : \mathbf{E}}{H + \omega \gamma^2}$$

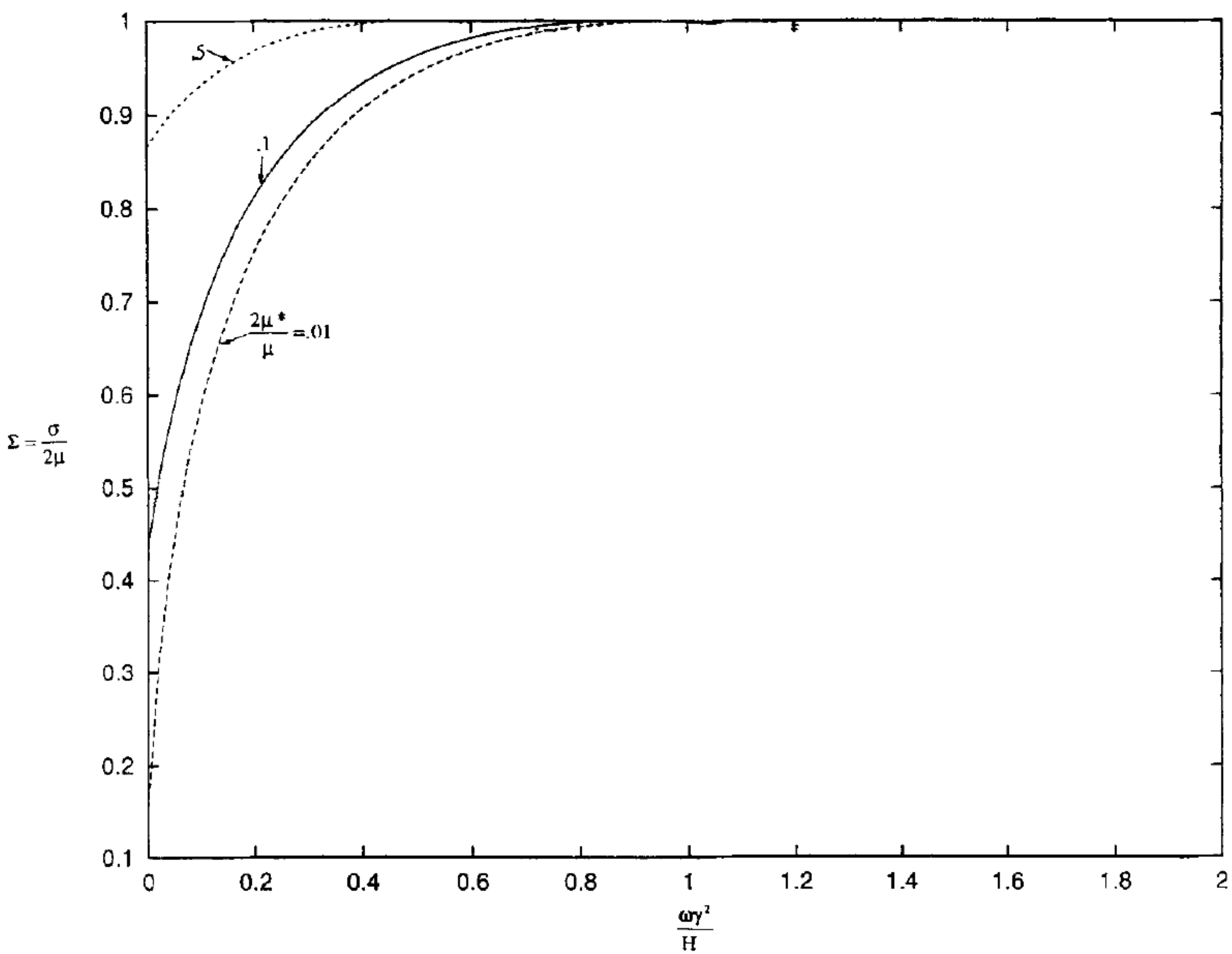
$$H + \omega \gamma^2 = \left(\mathbf{n} \cdot \frac{\partial f}{\partial \sigma} : \mathbf{E} \right) \cdot [\mathbf{n} \cdot (\mathbf{E} + \mathbf{A}) \cdot \mathbf{n}]^{-1} \cdot \left(\mathbf{n} \cdot \mathbf{E} : \frac{\partial F}{\partial \sigma} \right)$$

$$\omega \gamma^2 = H_{\mathrm{c}} - H$$

$$\psi(x_1, x_2) = \psi_0 \exp\left[i\gamma(n_1 x_1 + n_2 x_2)\right]$$

$$\lambda(x_1, x_2) = \lambda_0 \exp\left[i\gamma(n_1 x_1 + n_2 x_2)\right],$$

$$\hat{g}(x_1, x_2) = \hat{g}_0 \exp\left[i\gamma(n_1 x_1 + n_2 x_2)\right],$$



Modes de surface

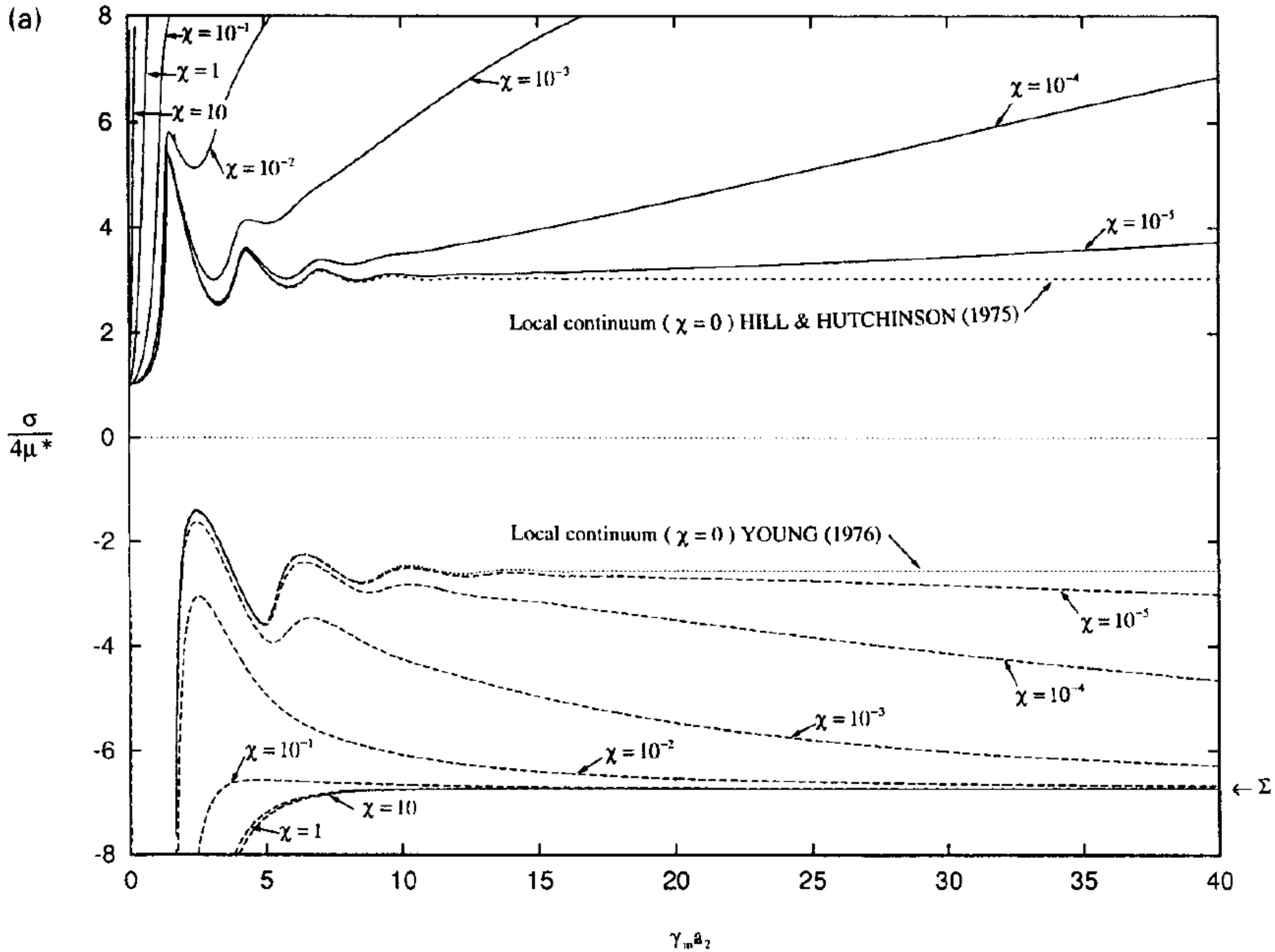
$$\gamma_m a_2 \to \infty$$

$$\Sigma^3 - 2\Sigma^2 + 2 = 0.$$

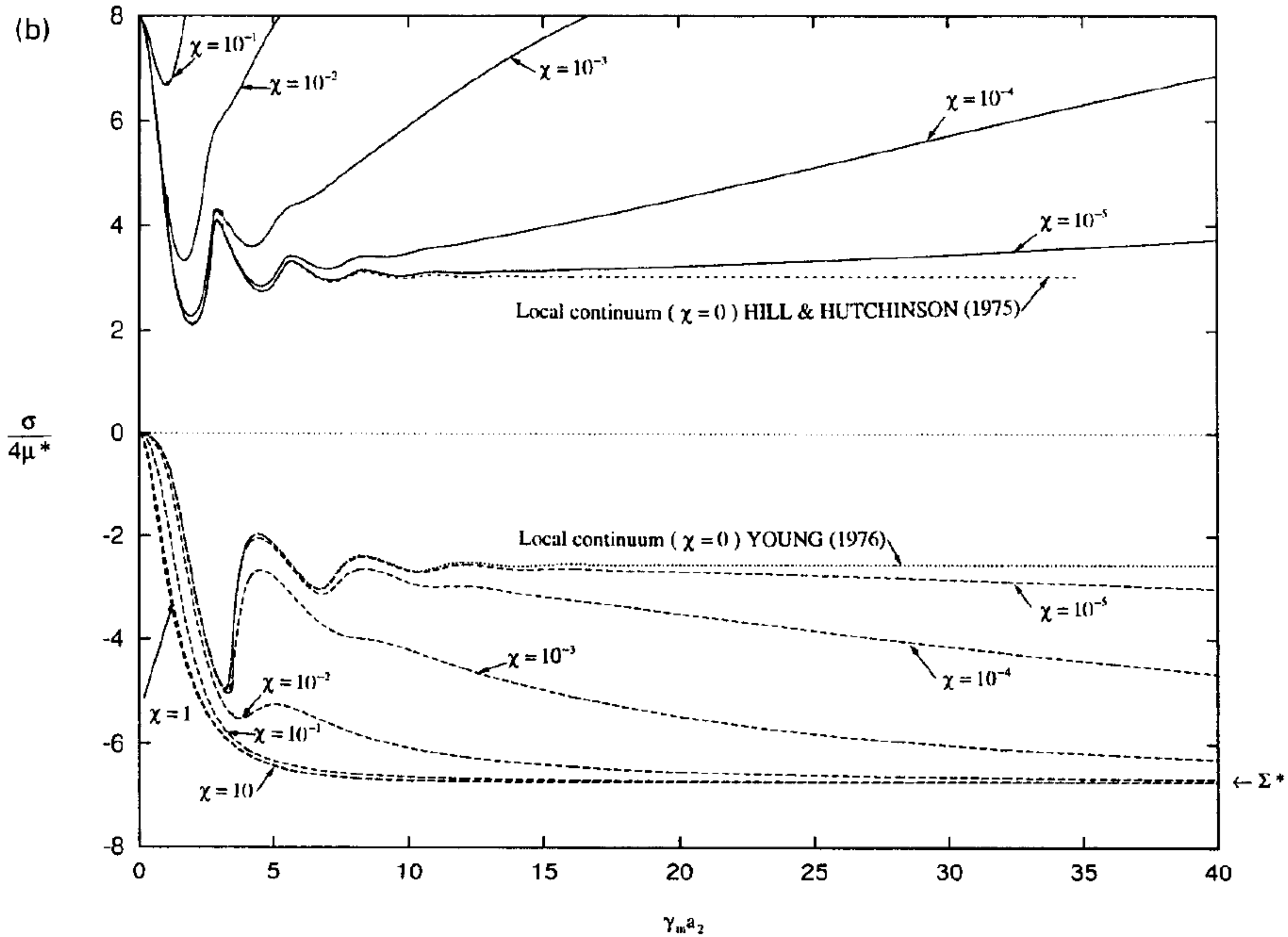
$$\frac{\sigma}{4\mu^*} = 1 + \frac{\sigma}{4\mu^*}\sqrt{\frac{2\mu-\sigma}{2\mu+\sigma}}$$

$$\Sigma^* = \tfrac{2}{3} + \tfrac{4}{3}\{-19+\sqrt{297}\}^{-1/3} + \tfrac{1}{3}\{-19+\sqrt{297}\}^{1/3} \approx -0.84$$

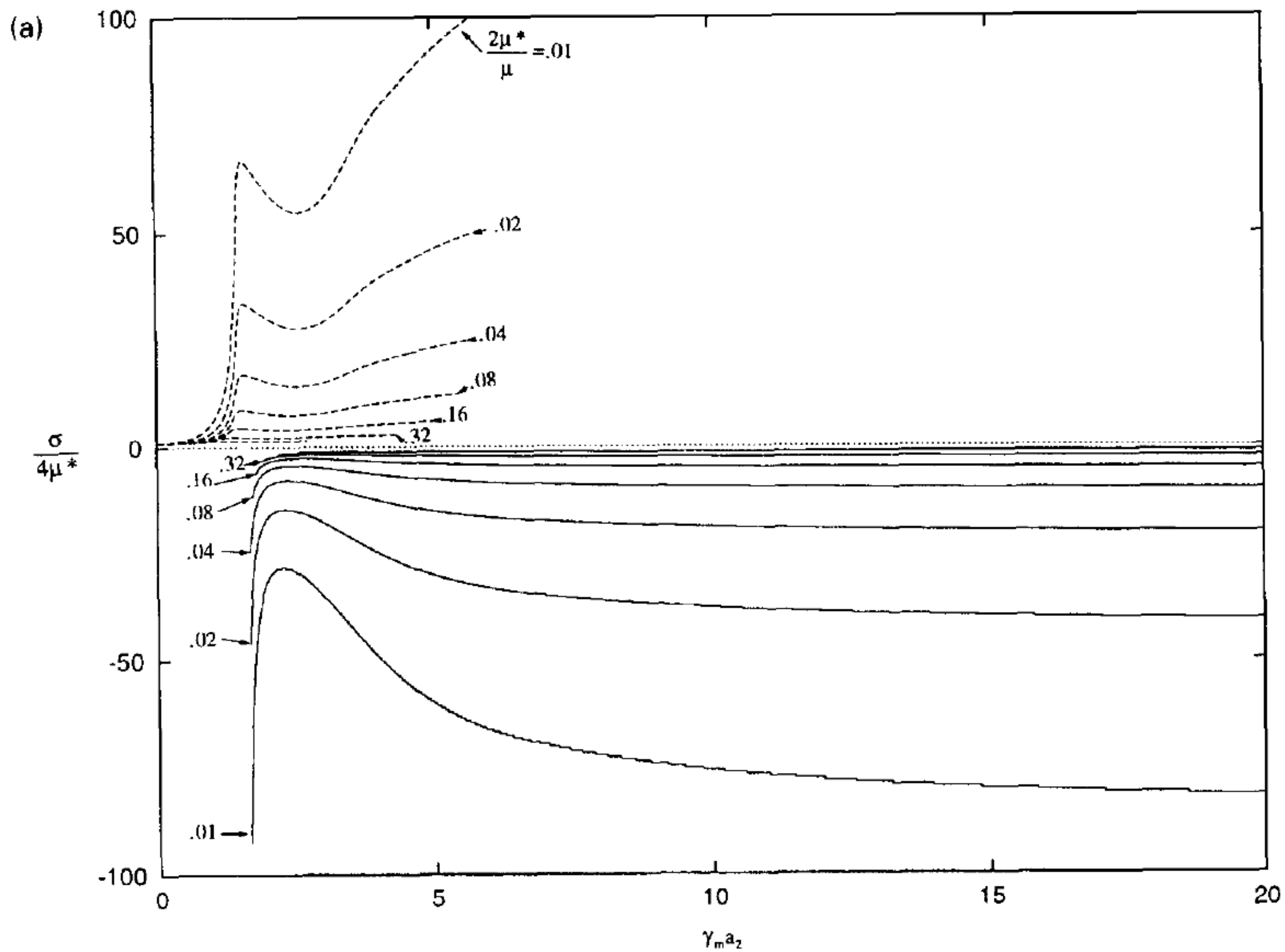
Uniquement en compression



$$2\mu^*/\mu = \frac{1}{8}.$$

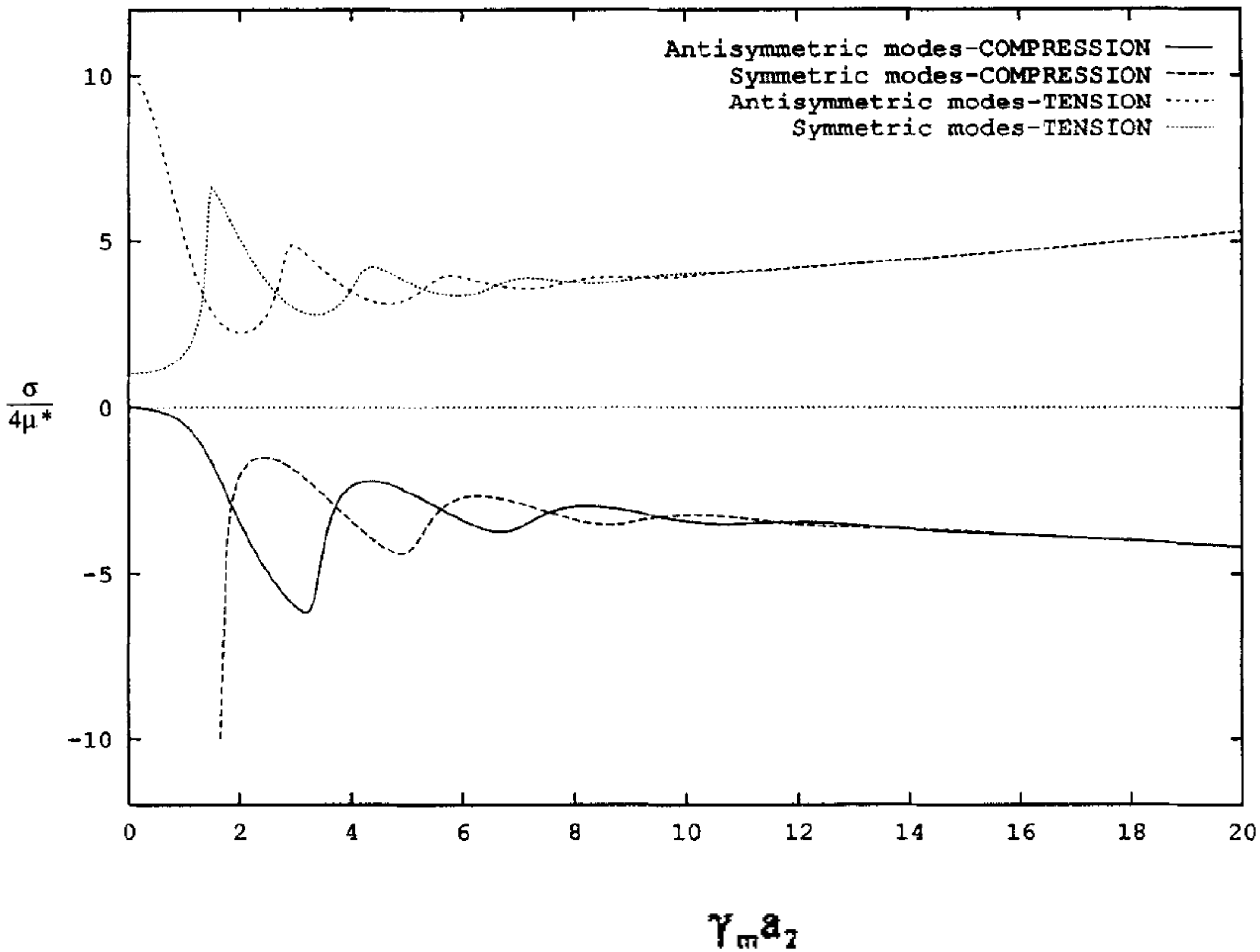


$$2\mu^*/\mu = \frac{1}{8}.$$



(a)
100
50
0
-50
-100
σ / 4μ*
0
5
10
15
20
γ_m a_2
2μ* / μ =.01
.02
.04
.08
.16
.32
.32
.16
.08
.04
.02
.01

$$\chi = 10^{-2}$$



$$2\mu^*/\mu = 0.1$$

$$\chi = 10^{-2}$$